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The capital asset pricing model and the liquidity effect: A theoretical approach☆

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Abstract

In this paper we develop a CAPM-based model to demonstrate that the true measure of systematic risk – when considering liquidity costs – is based on net (after bid–ask spread) returns. We further examine the relationship between the expected return and the future spread cost within the CAPM framework. This positive relationship in our model is found to be convex. This finding differs from Amihud and Mendelson's (1986) concave relationship, but it agrees with empirical evidence obtained by Brennan and Subrahmanyam (1996). © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

The capital asset pricing model (CAPM), derived by Sharpe (1964), Lintner (1965) and Mossin (1966) and in its zero-beta version by Black (1972), has a history of more than thirty years over four decades. While the CAPM received early empirical support, it was subsequently challenged on the basis of incompleteness.[1] Recently, the Fama and French (1992) study sparked a debate regarding the validity of the CAPM.[2] A number of papers attempted to address the incompleteness issue, notably Amihud and Mendelson (1986), Merton (1987) and Amihud and Mendelson (1989).

In their seminal paper, Amihud and Mendelson (1986) develop a model that positively relates returns to the relative spread (bid–ask spread divided by price) at a decreasing rate (a concave relationship). They empirically test the implications of their theoretical model using a CAPM framework. In separate regressions they find a linear relationship between excess returns and beta and confirm a concave relationship between excess returns and relative spread.[3] Amihud and Mendelson (1989) show that three of four factors identified by Merton (1987) as significantly related to risk-adjusted returns are no longer significant when the relative bid–ask spread is included as an explanatory variable.[4] Only beta remains significant. They conclude, therefore, that expected asset returns are positively related to beta and relative bid–ask spread.

Further empirical evidence of the importance of the bid–ask spread is provided by Datar et al. (1998) who, using data from the NYSE, show that trading volume explains returns better than the size of the firm.[5] They conclude that the size-return relationship reported by Fama and French (1992) is a reflection of the liquidity-return relationship, with size simply one of a number of possible surrogates for liquidity.[6]

[1] Empirical support includes Black et al. (1972) and Fama and MacBeth (1973). See Litzenberger and Ramaswamy (1979), Litzenberger and Ramaswamy (1980), and Banz (1981) for challenges on the basis of incompleteness.

[2] Fama and French (1992) claim that the CAPM has no explanatory power regarding the cross-sectional expected returns, while size and book-to-market ratio have an important role. This study has been challenged on the basis of statistical methodology, periodic effects, and the data used. See Black (1993), Kenz and Ready (1997), and Kothari et al. (1995).

[3] While Amihud and Mendelson's (1986) theoretical model is about riskless assets, they proceed to test its implications on risky assets.

[4] The factors are (1) β of the security; (2) firm size; (3) availability of information about the security (proxied by the fraction of investors who purchase it); and (4) residual risk caused by the application of the traditional CAPM.

[5] As well, they note that the trading volume-return relationship is observed in non-January months, while the size-return relationship is not.

[6] They further confirm this conclusion by demonstrating that another proxy for liquidity, number of shares outstanding, explains returns better than size as well.

Brennan and Subrahmanyam (1996) examine whether illiquidity costs caused by adverse selection results in higher expected returns. Instead of using bid–ask spreads as a proxy for liquidity they use estimated variable and fixed, proportional, transaction costs.[7] They adjust for risk using Fama and French's (1993) three factor model, not the CAPM, and find a concave relationship between premiums and variable costs and a convex relationship between premiums and fixed costs. This convexity is in contrast to the concave relationship found by Amihud and Mendelson (1986).[8]

Brennan et al. (1998) investigate whether expected returns are explained by a number of firm characteristics including market liquidity, measured using trading volume.[9] They utilize two risk adjustment measures, based on Connor and Korajcyzk (1988) and Fama and French (1993). They find a negative and significant relationship between returns and trading volume for both NYSE and NASDAQ stocks. They argue that since volume is significant using either risk adjustment method, support is provided for the hypothesis that there is an actual relationship between expected returns and liquidity, and incomplete risk adjustment is not a factor. The results of the papers cited above provide compelling evidence that the size effect is primarily a surrogate for the liquidity effect.

In this paper we derive a liquidity-adjusted version of the CAPM based on returns calculated after taking into account the effect of the bid–ask spread. Our model demonstrates that the measure of systematic risk should incorporate liquidity costs (the bid–ask spread).

The contribution of this paper is to demonstrate that beta and liquidity are inseparable. We develop a CAPM-based model by adopting Amihud and Mendelson's (1989) conclusion that the bid–ask spread is the true reason for the existence of the size effect. Our model shows that the true measure of systematic risk, when considering liquidity costs, has to be the one based on net after-spread returns. This theoretical conclusion anticipates that the beta measure and the spread effect are inseparable. By identifying a significant size effect, described by Fama and French (1992), with the spread effect, we suggest that an after-spread beta may produce significant results for the same period (1963–1990).

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[7] They argue that bid–ask spreads are not an appropriate proxy for liquidity as bid–ask spreads are noisy. As well, liquidity costs caused by asymmetric information are captured in the variable component of trading costs.

[8] Brennan and Subrahmanyam (1996) note that this inconsistency may possibly be due to inaccurately estimated parameters, or it may be an indication that the three factor model does not completely adjust for risk.

[9] They do not use the bid–ask spread as a measure of liquidity as it was unavailable in appropriate form in their data set. They further argue that trading volume is a superior measure of liquidity.

The after-spread beta measure we derive is non-linear in the traditional beta. The non-linear specification indicates that rejection of the traditional CAPM is expected, especially when the liquidity effect is significant. This point allows us to contrast the early empirical success of the CAPM obtained by Black et al. (1972), and Fama and MacBeth (1973) against the Fama and French (1992) study. The earlier studies only used data from the highly liquid NYSE, while the data used by Fama and French (1992) also includes securities from the less liquid AMEX and NASDAQ.

This supposition is further supported by another important result obtained by Kothari et al. (1995), who claim that when betas are estimated annually, a significant relationship is found for the periods 1927–1990, and 1941–1990. This result contradicts Fama and French's (1992) rejection of the CAPM for the same period (1941–1990) based on monthly estimation of the betas. These contradictory results can be explained by the fact that liquidity costs proxied by the bid–ask spread are more prominent for shorter (monthly) holding periods, while their relative importance weakens for longer (annual) holding periods.[10]

We further examine the relationship between the expected return and the future spread cost within the CAPM framework. This positive relationship in our model is found to be convex. This finding differs from Amihud and Mendelson's (1986) concave relationship, but it agrees with empirical evidence obtained by Brennan and Subrahmanyam (1996).

The remainder of this paper is organized as follows: Section 2 outlines the CAPM-based model incorporating the liquidity effect. Section 3 discusses the implications of our model. Summary and conclusions are offered in Section 4.

2. The modelling

In this section we develop the liquidity-adjusted CAPM. We assume a one-period world with an uncertain future bid–ask spread, where the security's measure of systematic risk incorporates the after-spread returns rather than gross returns. All the standard assumptions of the traditional CAPM are retained except for the market imperfection assumption and the joint normality of after-spread asset returns.[11]

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[10] Kothari et al. (1995) fail to support the CAPM for the period 1963–1990, which is consistent with Fama and French (1995). The two papers further agree about the existence of a significant size effect.

[11] That is, it is assumed that investors are risk-averse, expected utility maximizers of their end of period wealth. They all have homogenous expectations of asset returns, and behave as price takers. Investors can borrow and lend unlimited amounts at the risk free rate. It is also assumed that all assets are infinitely divisible and the quantities of assets are fixed.

Further, we add assumptions regarding the measure of liquidity. Assets have different levels of marketability and therefore transacting these assets involves liquidity costs that are measured by the relative spread. The asset's level of marketability varies over time so that the end of period bid–ask spread is a random variable. Investors have homogenous expectations about the end of period spread, and they cannot affect the liquidity level of an asset (price takers with respect to the spread). There are no liquidity costs for the risk-free asset. It is also assumed that risk-averse market makers stand by to provide immediacy services to the investors. In a competitive market, the market makers charge the bid–ask spread for providing these services. Finally, asset returns net of liquidity costs are jointly normally distributed, or investors and market makers have a quadratic utility function.

The representative investor's portfolio selection problem is:

$$\max_{B,V_j} \{\mathrm{E}[u_i(\tilde{Y}_i)]\} \tag{1}$$

subject to

$$W_i = B + \sum_{j=1}^{N} V_j(1 + S_j), \tag{2}$$

$$\tilde{Y}_i = R_f B + \sum_{j=1}^{N} V_j \tilde{R}_j(1 - \tilde{S}_j), \tag{3}$$

where

u_i = the utility function of investor $i, i = 1, 2, \ldots, I$, such that $u_i'(\cdot) > 0$, and $u_i''(\cdot) < 0$ (risk aversion)
W_i = investor i's initial wealth
$\tilde{Y}_i$ = investor i's terminal wealth
B = dollar amount invested in a risk-free asset
V_j = dollar amount invested in a risky security j, $j = 1, 2, \ldots, N$
$\tilde{R}_j$ = return (one plus rate of return) on risky security j, before liquidity costs (a random variable)
R_f = risk-free return (one plus rate of return) on the risk-free asset
S_j = the liquidity cost of transacting asset j, given by: $S_j = (a_j - P_j)/P_j = (P_j - b_j)/p_j$
where
a_j = the ask price of security j
b_j = the bid price of security j
P_j = the mid-price, given by: $(a_j + b_j)/2$
$\tilde{S}_j$ = the end of period liquidity cost of transacting asset j, (a random variable).

Constraint (2) is the investor's budget constraint. The second term on the right-hand side of (2) is the amount of money invested in risky assets including the spread costs. Constraint (3) is investor i's terminal wealth, which is given by the dollar return on the risk-free asset, plus the dollar return from the risky portfolio net of liquidity costs. Note that an investor can hold an asset long or short. In a short position, we simply have $V_j < 0$, $S_j < 0$, and $\tilde{S}_j < 0$.

Plugging the constraints into problem (1), we get

$$\max_{V_j} \left\{ \mathrm{E}\left[u_i\left(R_{\mathrm{f}}\left[W_i - \sum_{j=1}^{N} V_j(1+S_j) \right] + \sum_{j=1}^{N} V_j \tilde{R}_j (1-\tilde{S}_j) \right) \right] \right\}. \tag{4}$$

The first order condition for (4) is

$$\mathrm{E}[u_i'(\cdot)(-R_{\mathrm{f}}(1+S_j) + \tilde{R}_j(1-\tilde{S}_j))] = 0$$

which can be written as

$$\mathrm{E}[u_i'(\cdot)]\mathrm{E}[\tilde{R}_j(1-\tilde{S}_j) - R_{\mathrm{f}}(1+S_j)] + \mathrm{COV}(u_i'(\cdot), \tilde{R}_j(1-\tilde{S}_j)) = 0. \tag{5}$$

Since returns net of spread cost are jointly normal, (5) yields:

$$\mathrm{E}[u_i'(\cdot)]\mathrm{E}[\tilde{R}_j(1-\tilde{S}_j) - R_{\mathrm{f}}(1+S_j)] + \mathrm{E}[u_i''(\cdot)]\,\mathrm{COV}(\tilde{Y}_i, \tilde{R}_j(1-\tilde{S}_j)) = 0.$$

Let $\theta_i = -\mathrm{E}[u_i'(\cdot)]/\mathrm{E}[u_i'(\cdot)]$ for investor i, then

$$\theta_i \mathrm{E}[\tilde{R}_j(1-\tilde{S}_j) - R_{\mathrm{f}}(1+S_j)] = \mathrm{COV}(\tilde{Y}_i, \tilde{R}_j(1-\tilde{S}_j)). \tag{6}$$

Let $\theta = \sum_{i=1}^{I} \theta_i$ and $\tilde{Y} = \sum_{i=1}^{I} \tilde{Y}_i$, and sum equation (6) over all investors,

$$\theta \mathrm{E}[\tilde{R}_j(1-\tilde{S}_j) - R_{\mathrm{f}}(1+S_j)] = \mathrm{COV}(\tilde{Y}, \tilde{R}_j(1-\tilde{S}_j)). \tag{7}$$

Define (7) for the market portfolio

$$\theta \mathrm{E}\left[\sum_j (\tilde{R}_j(1-\tilde{S}_j)\omega_j) - R_{\mathrm{f}}(1+S_m) \right] = \mathrm{COV}\left(\tilde{Y}, \sum_j (\tilde{R}_j(1-\tilde{S}_j)\omega_j) \right) \tag{8}$$

where $S_m = \sum_j S_j \omega_j$, and ω_j is asset j's relative value in the market portfolio.

Now consider $\sum_j (\tilde{R}_j(1-\tilde{S}_j)\omega_j)$, which can be written as: $\tilde{R}_m - \sum_j \tilde{R}_j \tilde{S}_j \omega_j$, where $\tilde{R}_m = \sum_j \tilde{R}_j \omega_j$. And

$$\sum_j \tilde{R}_j \tilde{S}_j \omega_j = \sum_j \frac{\tilde{P}_j}{P_j} \tilde{S}_j \frac{P_j}{\sum_j P_j} = \frac{\sum_j \tilde{a}_j - \tilde{P}_j}{\sum_j P_j} \equiv \tilde{C}_m,$$

where $\tilde{C}_m$ is the dollar amount spent in liquidity costs at the end of the period, relative to the value of the market portfolio today. We rewrite (8) as

$$\theta \mathrm{E}[(\tilde{R}_m - \tilde{C}_m) - R_{\mathrm{f}}(1+S_m)] = \mathrm{COV}(\tilde{Y}, (\tilde{R}_m - \tilde{C}_m)). \tag{9}$$

Divide (7) by (9) and note that $\tilde{Y}$ is linear in $(\tilde{R}_m - \tilde{C}_m)$:

$$\frac{\mathrm{E}[\tilde{R}_j(1-\tilde{S}_j) - R_\mathrm{f}(1+S_j)]}{\mathrm{E}[(\tilde{R}_m - \tilde{C}_m) - R_\mathrm{f}(1+S_m)]} = \frac{\mathrm{COV}(\tilde{Y}, \tilde{R}_j(1-\tilde{S}_j))}{\mathrm{COV}(\tilde{Y}, (\tilde{R}_m - \tilde{C}_m))}$$

$$= \frac{\mathrm{COV}((\tilde{R}_m - \tilde{C}_m), \tilde{R}_j(1-\tilde{S}_j))}{\mathrm{VAR}(\tilde{R}_m - \tilde{C}_m)}.$$

Rearranging, we get the expected net (after-spread) return:

$$\mathrm{E}\left[\tilde{R}_j\frac{(1-\tilde{S}_j)}{(1+S_j)}\right]$$

$$= R_\mathrm{f} + \frac{\mathrm{COV}((\tilde{R}_m - \tilde{C}_m), \tilde{R}_j(1-\tilde{S}_j))}{\mathrm{VAR}(\tilde{R}_m - \tilde{C}_m)}\mathrm{E}[(\tilde{R}_m - \tilde{C}_m) - R_\mathrm{f}(1+S_m)]$$

or

$$\mathrm{E}\left[\tilde{R}_j\frac{(1-\tilde{S}_j)}{(1+S_j)}\right] = R_\mathrm{f} + \frac{\mathrm{COV}\left(\frac{(\tilde{R}_m - \tilde{C}_m)}{(1+S_m)}, \tilde{R}_j\frac{(1-\tilde{S}_j)}{(1+S_j)}\right)}{\mathrm{VAR}\left(\frac{(\tilde{R}_m - \tilde{C}_m)}{(1+S_m)}\right)} \times \left\{\mathrm{E}\left[\frac{(\tilde{R}_m - \tilde{C}_m)}{(1+S_m)}\right] - R_\mathrm{f}\right\}. \tag{10}$$

This is the expected net (after-spread) return on asset j, given by the risk-free return plus asset j's measure of systematic risk times the net (after-spread) market risk premium. Note that when the end of period spread ratio is unknown, the investor bears an additional risk of adverse changes in the spread level, and is thus interested in the systematic risk measure based on the net returns (after-spread returns) rather than the traditional (pre-spread) beta. Let

$$\mathrm{E}\left[\tilde{R}_j\frac{(1-\tilde{S}_j)}{(1+S_j)}\right] \equiv \mathrm{E}[\tilde{R}_j^*], \qquad \mathrm{E}\left[\frac{(\tilde{R}_m - \tilde{C}_m)}{(1+S_m)}\right] \equiv \mathrm{E}[\tilde{R}_m^*]$$

and

$$\frac{\mathrm{COV}\left(\frac{(\tilde{R}_m - \tilde{C}_m)}{(1+S_m)}, \tilde{R}_j\frac{(1-\tilde{S}_j)}{(1+S_j)}\right)}{\mathrm{VAR}\left(\frac{(\tilde{R}_m - \tilde{C}_m)}{(1+S_m)}\right)} \equiv \beta_j^*.$$

Then (10) can be rewritten as

$$\mathrm{E}[\tilde{R}_j^*] = R_\mathrm{f} + \beta_j^*[\mathrm{E}[\tilde{R}_m^*] - R_\mathrm{f}]. \tag{11}$$

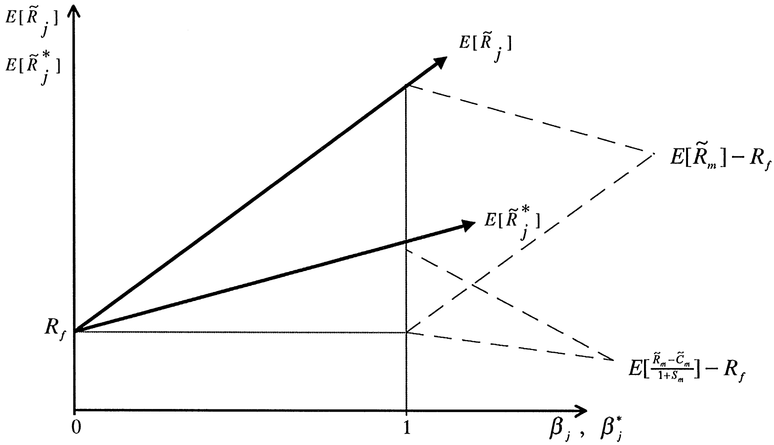

Fig. 1. The Relationship between the expected return and the measure of systematic risk. $E[\tilde{R}_j]$ and $E[\tilde{R}_j^*]$ are the asset expected gross returns given by the CAPM and our modified model respectively. R_f is the return on the risk-free asset. β_j and β_j^* are the measures of systematic risk implied by the CAPM and our modified model respectively. Our modified model suggests that the risk premium per unit of systematic risk is lower than that suggested by the traditional CAPM. The difference in the slopes is positive, since $E[\tilde{R}_m] > E[\tilde{R}_m^*]$.

The traditional CAPM is given by

$$E[\tilde{R}_j] = R_f + \beta_j[E[\tilde{R}_m] - R_f], \tag{12}$$

where

$$\beta_j = \frac{COV(\tilde{R}_m, \tilde{R}_j)}{VAR(\tilde{R}_m)}.$$

Fig. 1 describes the relationship between the expected return and the measure of systematic risk in both models. Note that the intercept under both models is the risk-free return since we assumed zero spread for the risk-free asset.[12] However, the two models differ in the slope. The modified model suggests that the risk premium per unit of systematic risk is lower than that suggested by the traditional CAPM. The difference in the slopes is positive, since $E[\tilde{R}_m] > E[\tilde{R}_m^*]$.[13]

[12] This assumption is very reasonable since for instance US Treasury bills have a spread ratio close to zero.

[13] We say that $E[\tilde{R}_m]$ is strictly greater than $E[\tilde{R}_m^*]$ since it is unlikely that the market portfolio will consist of only extremely liquid securities with zero liquidity costs.

Eq. (10) can be represented in terms of expected gross (pre-spread) return:

$$\mathrm{E}[\tilde{R}_j] = \left\{ R_{\mathrm{f}} \frac{(1+S_j)}{(1-\mathrm{E}[\tilde{S}_j])} + \frac{\mathrm{COV}(\tilde{R}_j, \tilde{S}_j)}{(1-\mathrm{E}[\tilde{S}_j])} \right\} + \beta_j^* \frac{(1+S_j)}{(1-\mathrm{E}[\tilde{S}_j])} \left\{ \mathrm{E}\left[\frac{(\tilde{R}_m - \tilde{C}_m)}{(1+S_m)} \right] - R_{\mathrm{f}} \right\}. \quad (13)$$

At this point, for the purpose of comparative statics, we assume that $\sum_j \tilde{a}_j - \tilde{P}_j$ is a martingale, so that

$$\mathrm{E}[\tilde{C}_m] = \frac{\mathrm{E}[\sum_j \tilde{a}_j - \tilde{P}_j]}{\sum_j P_j} = \frac{\sum_j a_j - P_j}{\sum_j P_j} = \sum_j S_j \frac{P_j}{\sum_j P_j} = \sum_j S_j \omega_j = S_m.$$

This assumption is reasonable since on average, the market's spread is not expected to change over the one period interval. The martingale assumption also applies for securities of companies with large market values, which are generally very liquid and their spread is constant on average. For other securities including illiquid assets, one cannot expect the spread to be on average constant over time. Therefore, we assume that the spread for these securities will follow a sub-martingale or a super-martingale such that overall, the sum of all end of period spreads in the market portfolio, $\sum_j \tilde{a}_j - \tilde{P}_j$ follows a martingale.

The martingale assumption implies that the expected end of period spread costs for the market portfolio is insensitive to changes in the expected end of period spread ratio for the single security. Formally, this means that $\partial \mathrm{E}[\tilde{C}_n]/\partial \mathrm{E}[\tilde{S}_j] = 0$, and it enables us to show that the first and second derivatives of the expected gross return of assets with respect to its expected end of period spread, are both positive. Thus, the expected gross return is increasing and convex in the expected spread ratio. This result is different from the finding by Amihud and Mendelson (1986), who suggest that the expected gross return is increasing and concave in the next period spread. They point out that heavily traded securities are likely to be traded more frequently in the future. Therefore, for a given increase in the level of the spread ratio, these securities will have a higher value of additional amortized spread costs, relative to thinly traded securities. For that reason, investors will demand a higher liquidity premium for a given change in the spread, for the more active assets. Amihud and Mendelson call this effect *the clientele effect*.

3. Implications of our model

The CAPM based model derived in this paper is a one period model, under which all securities, liquid and illiquid, are held for the whole period. This means that we do not allow the more liquid asset to be traded more frequently during

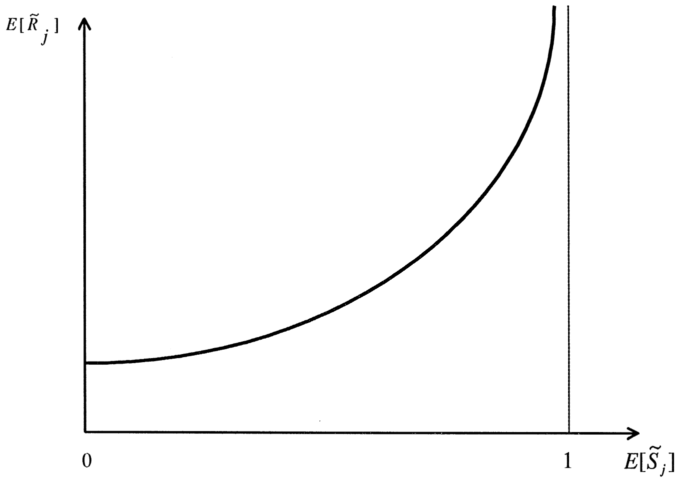

Fig. 2 . The relationship between the expected gross return and the expected future spread. $E[\tilde{R}_j]$ and $E[\tilde{S}_j]$ are the asset expected gross returns and expected end of period spread ratio given by our modified model respectively. As the expected end of period spread ratio approaches 1, the investor will demand an infinite compensation in terms of expected gross return. Thus, the expected return grows to infinity asymptotically to the vertical line $E[\tilde{S}_j] = 1$, and therefore it must be convex in the expected spread ratio.

the period, thereby eliminating the clientele effect and the concavity obtained by Amihud and Mendelson.

Although this is a shortcoming, the model still emphasizes two important issues not addressed in Amihud and Mendelson's (1986) framework. First, the convexity demonstrated by our model has to hold for securities with high spreads. As the expected end of period spread approaches 1 (100%), the investor will demand an infinite compensation in terms of expected gross return before entering a long position in such an asset. This means that the expected return grows to infinity asymptotically to the vertical line $E[\tilde{S}_j] = 1$, and therefore it must be convex in the expected spread for high spread levels (see Fig. 2).[14] We call the above effect the *level effect*.

Thus, this model, together with Amihud and Mendelson (1986), gives rise to the two opposite effects determining the curvature of the function relating the expected gross return to the future relative spread. Empirical evidence presented by Brennan and Subrahmanyam's (1996), confirms the above theoretical result.

[14] For a short position, as $E[S_j] \to 1$, the investor will have to pay 100% of the security value as liquidity cost at the closing of the position. This is high enough to allow us to maintain the above argument for a short position as well.

They find that investors demand a positive return premium for the cost of liquidity. Brennan and Subrahmanyam decompose the estimated trading cost into variable and fixed components. Their empirical results support a concave relationship between the variable cost of transacting and the return premium. This is consistent with Amihud and Mendelson's (1986) clientele effect. Brennan and Subrahmanyam also find a convex relationship between the fixed cost component and the return premium. Noting that they find this fixed cost component to be highly correlated with the relative spread (correlation of 0.78), the last result seems to agree with the level effect presented above.[15]

The second factor our model emphasizes is apparent from the after-spread beta. As demonstrated above, the measure of systematic risk must account for possible adverse changes in the level of the security's spread ratio. Brennan and Subrahmanyam (1996) show that the spread ratio is negatively related to excess returns. They hypothesize that this counterintuitive result is due to the spread ratio proxying for a risk measure which is not captured by the Fama and French (1993) three-factor model they use to adjust for risk. The model derived above demonstrates that beta and liquidity measured by the spread ratio are inseparable. Thus, when one adjusts for risk, one should not ignore the impact of uncertainty related to the spread ratio on the assets systematic risk. This supports Brennan and Subrahmanyam's assertion regarding their counterintuitive result.

Amihud and Mendelson (1986) provide empirical evidence supporting their model. The data used for the empirical test is sampled from the NYSE for the years 1960–1979. The highest average of the beginning and end-of-year spread ratio reported in the paper is less than 3.5%. Therefore, it is reasonable that for a market as liquid as the NYSE – where securities are traded with high frequency – Amihud and Mendelson's clientele effect will dominate our level effect, and thus an empirical test based on this market alone may lead to a conclusion that the relationship between the expected gross return and the spread ratio is concave over the entire spectrum of feasible spread ratios.

4. Summary and conclusion

In this paper we examined the relationship between the expected gross return required by investors and the level of the relative spread. A CAPM-based

[15] Brennan and Subrahmanyam (1996) suggest two possible reasons for the convexity: (i) inaccurate estimation of the fixed cost component measure, and (ii) incomplete risk adjustment by the three-factor Fama and French (1993) model they use.

model is derived to show that the measure of systematic risk in a world with uncertainty with regard to the future spread, is one calculated on the basis of the net (after-spread) returns. Identifying the size effect with the spread effect, we suggested that rejection of the traditional CAPM is expected, since the after-spread beta measure is non-linear in the traditional beta. This supposition is supported by existing empirical evidence. The model in this paper yields a result different from that of a model derived by Amihud and Mendelson (1986). Amihud and Mendelson's model suggests a positive and concave relationship between the expected gross return and the future spread. Our model predicts a convex relationship instead. This theoretical result, and the conclusion that the measure of systematic risk must account for possible adverse changes in the level of the security's spread ratio, are supported by empirical evidence by Brennan and Subrahmanyam (1996).

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